Grupa A - Pismeni ispit iz Matematike, 13.06.2013.
ispit pisati isključivo hemiskom olovkom

1. Vektor $v \in \mathbb{R}$ u odnosu na bazu $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 2 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -3 \end{bmatrix} \right\}$ ima koordinate $\begin{bmatrix} 4 \\ -1 \\ 7 \end{bmatrix}$. Odrediti koordinate vektora v u odnosu na bazu $\mathcal{B}' = \left\{ \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\}$.

2.
(60%) (a) Odrediti ekstreme, prevojne tačke te intervale konveksnosti i konkavnosti funkcije

$$y = \frac{e^{2x}}{x+1}$$

(40%) (b) Odrediti parametre a i b tako da je prava $y = x - 4$ kosa asimptota funkcije

$$y = \frac{(ax+b)^4}{x^3}.$$

3. Odrediti ekstreme funkcije $z = x^2 + y^3 + 4x\sqrt{x^3} - 3y$.

4. Odrediti integral $\displaystyle\int \frac{\mathrm{d}x}{3x - 4\sqrt{x}}$.

Grupa B - Pismeni ispit iz Matematike, 13.06.2013.
ispit pisati isključivo hemiskom olovkom

1. Vektor $v \in \mathbb{R}$ u odnosu na bazu $\mathcal{B} = \left\{ \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} \right\}$ ima koordinate $\begin{bmatrix} 5 \\ -1 \\ 3 \end{bmatrix}$. Odrediti koordinate vektora v u odnosu na bazu $\mathcal{B}' = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$.

2.
(60%) (a) Odrediti ekstreme, prevojne tačke te intervale konveksnosti i konkavnosti funkcije

$$y = \frac{e^{3x}}{1 + e^{-x}}$$

(40%) (b) Odrediti parametre a i b tako da tako da je prava $y = 27x + 9$ kosa asimptota funkcije

$$y = \frac{(ax+b)^3}{x^2}.$$

3. Odrediti ekstreme funkcije $z = 3\ln\frac{x}{6} + \ln(12 - y - x) + 2\ln y$.

4. Odrediti integral $\displaystyle\int \frac{\sqrt[6]{x+1}\,\mathrm{d}x}{\sqrt{x+1} + \sqrt[3]{x+1}}$.

Grupa C - Pismeni ispit iz Matematike, 13.06.2013. ispit pisati isključivo hemiskom olovkom

1. Vektor $v \in \mathbb{R}$ u odnosu na bazu $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$ ima koordinate $\begin{bmatrix} 7 \\ -3 \\ 5 \end{bmatrix}$. Odrediti koordinate vektora v u odnosu na bazu $\mathcal{B}' = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$.

2.
(60%) (a) Odrediti ekstreme, prevojne tačke te intervale konveksnosti i konkavnosti funkcije

$$y = \frac{e^{2x}}{1 + e^{2x}}.$$

(40%) (b) Odrediti parametre a i b tako da je prava $y = 4x + 4$ kosa asimptota funkcije

$$y = \frac{(ax + b)^2}{x}.$$

3. Odrediti ekstreme funkcije $z = x^3 + y^2 - 3x + 4\sqrt{y^5}$.

4. Odrediti integral $\displaystyle\int \frac{\sqrt{x}\,\mathrm{d}x}{\sqrt[4]{x^3} + 4}$.

Grupa D - Pismeni ispit iz Matematike, 13.06.2013. ispit pisati isključivo hemiskom olovkom

1. Vektor $v \in \mathbb{R}$ u odnosu na bazu $\mathcal{B} = \left\{ \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} \right\}$ ima koordinate $\begin{bmatrix} 6 \\ -2 \\ 4 \end{bmatrix}$. Odrediti koordinate vektora v u odnosu na bazu $\mathcal{B}' = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$.

2.
(60%) (a) Odrediti ekstreme, prevojne tačke te intervale konveksnosti i konkavnosti funkcije

$$y = \frac{e^{3x}}{1 - x}$$

(40%) (b) Odrediti parametre a i b tako da je prava $y = 64x - 27$ kosa asimptota funkcije

$$y = \frac{a^2x^3 + b^3x^2 + 1}{x^2}.$$

3. Odrediti ekstreme funkcije $z = 2\ln x + \ln(12 - x - y) + 3\ln\dfrac{y}{6}$.

4. Odrediti integral $\displaystyle\int \frac{\sqrt{x}\,dx}{\sqrt[3]{x^2} + \sqrt[4]{x}}$.

Vektor $v \in \mathbb{R}^3$ u odnosu na bazu $\mathcal{B} = \left\{ \begin{pmatrix} 1 \\ 2 \\ 6 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 6 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ -3 \end{pmatrix} \right\}$ ima koordinate $\begin{pmatrix} 4 \\ -1 \\ 7 \end{pmatrix}$. Odrediti koordinate vektora v u odnosu na bazu $\mathcal{B}' = \left\{ \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \right\}$.

Rj.-upute.

Posmatrajmo baze $\mathcal{B}$ i $\mathcal{B}'$. Nije teško vidjeti da je

$$\left.\begin{aligned}
\begin{pmatrix} 1 \\ 2 \\ 6 \end{pmatrix} &= 1 \cdot \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} + 0 \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} + 1 \cdot \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \\
\begin{pmatrix} 0 \\ 2 \\ 6 \end{pmatrix} &= 1 \cdot \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} + 1 \cdot \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} + 0 \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \\
\begin{pmatrix} 0 \\ 0 \\ -3 \end{pmatrix} &= (-1) \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} + 0 \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} + 0 \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}
\end{aligned}\right\} \ldots (*)$$

Kako su koordinate vektora v u odnosu na bazu $\mathcal{B}$ $\begin{pmatrix} 4 \\ -1 \\ 7 \end{pmatrix}$ to znači da je $v = 4 \begin{pmatrix} 1 \\ 2 \\ 6 \end{pmatrix} - 1 \begin{pmatrix} 0 \\ 2 \\ 6 \end{pmatrix} + 7 \begin{pmatrix} 0 \\ 0 \\ -3 \end{pmatrix}$.

Prema $(*)$ imamo

$$4 \begin{pmatrix} 1 \\ 2 \\ 6 \end{pmatrix} = 4 \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} + \qquad + 4 \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$(-1) \begin{pmatrix} 0 \\ 2 \\ 6 \end{pmatrix} = (-1) \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} + (-1) \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}$$

$$7 \begin{pmatrix} 0 \\ 0 \\ -3 \end{pmatrix} = (-7) \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix}$$

Prema tome $v = (-4) \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} - 1 \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} + 4 \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$.

Koordinate vektora v u odnosu na bazu $\mathcal{B}'$ su $\begin{pmatrix} -4 \\ -1 \\ 4 \end{pmatrix}$.

\# Vektor $v \in \mathbb{R}^3$ u odnosu na bazu $\mathcal{B}=\left\{\begin{pmatrix}2\\1\\2\end{pmatrix},\begin{pmatrix}0\\-1\\1\end{pmatrix},\begin{pmatrix}0\\0\\2\end{pmatrix}\right\}$ ima koordinate $\begin{pmatrix}5\\-1\\3\end{pmatrix}$. Odrediti koordinate vektora v u odnosu na bazu $\mathcal{B}'=\left\{\begin{pmatrix}1\\1\\1\end{pmatrix},\begin{pmatrix}1\\0\\0\end{pmatrix},\begin{pmatrix}0\\0\\1\end{pmatrix}\right\}$.

Rj. - upute:

Posmatrajmo baze $\mathcal{B}$ i $\mathcal{B}'$. Nije teško vidjeti da je

$$\left.\begin{aligned}
\begin{pmatrix}2\\1\\2\end{pmatrix} &= 1\cdot\begin{pmatrix}1\\1\\1\end{pmatrix}+1\cdot\begin{pmatrix}1\\0\\0\end{pmatrix}+1\cdot\begin{pmatrix}0\\0\\1\end{pmatrix}\\
\begin{pmatrix}0\\-1\\1\end{pmatrix} &= (-1)\begin{pmatrix}1\\1\\1\end{pmatrix}+1\begin{pmatrix}1\\0\\0\end{pmatrix}+2\cdot\begin{pmatrix}0\\0\\1\end{pmatrix}\\
\begin{pmatrix}0\\0\\2\end{pmatrix} &= 0\cdot\begin{pmatrix}1\\1\\1\end{pmatrix}+0\begin{pmatrix}1\\0\\0\end{pmatrix}+2\begin{pmatrix}0\\0\\1\end{pmatrix}
\end{aligned}\right\}\ \ldots(*)$$

Kako su koordinate vektora $v\in\mathbb{R}^3$ u odnosu na bazu $\mathcal{B}$ $\begin{pmatrix}5\\-1\\3\end{pmatrix}$ to znači da je $v=5\begin{pmatrix}2\\1\\2\end{pmatrix}-1\begin{pmatrix}0\\-1\\1\end{pmatrix}+3\begin{pmatrix}0\\0\\2\end{pmatrix}$.

Prema $(*)$ imamo

$$\begin{aligned}
5\begin{pmatrix}2\\1\\2\end{pmatrix} &= 5\begin{pmatrix}1\\1\\1\end{pmatrix}+5\begin{pmatrix}1\\0\\0\end{pmatrix}+5\begin{pmatrix}0\\0\\1\end{pmatrix}\\
(-1)\begin{pmatrix}0\\-1\\1\end{pmatrix} &= 1\cdot\begin{pmatrix}1\\1\\1\end{pmatrix}+(-1)\begin{pmatrix}1\\0\\0\end{pmatrix}+(-2)\begin{pmatrix}0\\0\\1\end{pmatrix}\\
3\begin{pmatrix}0\\0\\2\end{pmatrix} &= 6\begin{pmatrix}0\\0\\1\end{pmatrix}
\end{aligned}$$

Prema tome $v=6\begin{pmatrix}1\\1\\1\end{pmatrix}+4\begin{pmatrix}1\\0\\0\end{pmatrix}+9\begin{pmatrix}0\\0\\1\end{pmatrix}$

Koordinate vektora v u odnosu na bazu $\mathcal{B}'$ su $\begin{pmatrix}6\\4\\9\end{pmatrix}$.

Vektor $v \in \mathbb{R}^3$ u odnosu na bazu $\mathcal{B} = \left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\}$ ima koordinate $\begin{pmatrix} 7 \\ -3 \\ 5 \end{pmatrix}$. Odrediti koordinate vektora v u odnosu na bazu $\mathcal{B}' = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$.

Rj.-upute:

Posmatrajmo baze $\mathcal{B}$ i $\mathcal{B}'$. Nije teško vidjeti da je

$$\left.\begin{array}{l}
\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 0 \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + 1 \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + 0 \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \\
\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = (-1) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + 0 \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + 1 \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \\
\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = 1 \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + (-1) \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + 1 \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}
\end{array}\right\} \ldots (*)$$

Kako su koordinate vektora $v \in \mathbb{R}^3$ u odnosu na bazu $\mathcal{B}$ $\begin{pmatrix} 7 \\ -3 \\ 5 \end{pmatrix}$ to znači da je $v = 7 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} - 3 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + 5 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$.

Sad prema $(*)$ imamo

$$7 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 7 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$-3 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = 3 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + (-3) \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$5 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = 5 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + (-5) \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + 5 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

Prema tome $v = 8 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + 2 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + 2 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

Koordinate vektora v u odnosu na bazu $\mathcal{B}'$ su $\begin{pmatrix} 8 \\ 2 \\ 2 \end{pmatrix}$.

(#) Vektor $v \in \mathbb{R}^3$ u odnosu na bazu $\mathcal{B} = \left\{ \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} \right\}$ ima koordinate $\begin{pmatrix} 6 \\ -2 \\ 4 \end{pmatrix}$. Odrediti koordinate vektora v u odnosu na bazu $\mathcal{B}' = \left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right\}$.

Rj.-upute:

Posmatrajmo baze $\mathcal{B}$ i $\mathcal{B}'$. Nije teško vidjeti da je

$$\left.\begin{aligned}
\begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} &= 1 \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + 1 \cdot \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + 1 \cdot \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \\
\begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix} &= 1 \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + (-1) \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + 2 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \\
\begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} &= (-1) \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + 1 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + 1 \cdot \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}
\end{aligned}\right\} \quad \ldots (*)$$

Kako su koordinate vektora $v \in \mathbb{R}^3$ u odnosu na bazu $\mathcal{B}$ $\begin{pmatrix} 6 \\ -2 \\ 4 \end{pmatrix}$ to znači da je $v = 6 \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} - 2 \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix} + 4 \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$.

Prema $(*)$ imamo

$$\begin{aligned}
6 \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} &= 6 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + 6 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + 6 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \\
-2 \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix} &= (-2) \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + 2 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + (-4) \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \\
4 \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} &= (-4) \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + 4 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + 4 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}
\end{aligned}$$

Prema tome $\quad v = 0 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + 12 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + 6 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

Koordinate vektora v u odnosu na bazu $\mathcal{B}'$ su $\begin{pmatrix} 0 \\ 12 \\ 6 \end{pmatrix}$.

Odrediti ekstreme, prevojne tačke te intervale konveksnosti i konkavnosti f-je $y = \dfrac{e^{2x}}{x+1}$.

Rj.-upute

D.p. $x+1 \neq 0$

$x \in (-\infty, -1) \cup (-1, +\infty)$

$$y' = \frac{e^{2x}(2x+1)}{(x+1)^2}$$

$y' = 0$ akko $2x+1=0$

$x = -\frac{1}{2}$

$-1 \quad -\frac{1}{2}$

← prekidi f-je y + nule f-je y'

x	$(-\infty, -1)$	$(-1, -\frac{1}{2})$	$(-\frac{1}{2}, +\infty)$
y'	$-$	$-$	$+$
y	$\searrow$	$\searrow$	$\nearrow$
		MIN	

tabela rasta i opadanja

$$f\left(-\frac{1}{2}\right) = \frac{e^{-1}}{\frac{1}{2}} = \frac{2}{e}$$

$$MIN\left(-\frac{1}{2}, \frac{2}{e}\right)$$

$$y'' = \frac{2e^{2x}(2x^2+2x+1)}{(x+1)^3}$$

$2x^2+2x+1=0$

$D = 4-8 < 0$

$2x^2+2x+1 > 0 \quad \forall x$

$y'' \neq 0 \quad \forall x \in \mathbb{R}$

-1

← prekidi y + nule y''

x	$(-\infty, -1)$	$(-1, +\infty)$
y''	$-$	$+$
y	$\frown$	$\smile$

tabela konveksnosti i konkavnosti

F-ja nema prevojnih tačaka.

(#) Odrediti ekstreme, prevojne tačke te intervale konveksnosti i konkavnosti f-je $y = \frac{e^{3x}}{1+e^{-x}}$.

Rj. -upute:

$1+e^{-x} \neq 0$ D. p. $x \in R$

$\underbrace{e^{-x}}_{\substack{>0 \\ \forall x}} \neq -1$ $x \in (-\infty, +\infty)$

$y' = \frac{e^{3x}(4e^{-x}+3)}{(1+e^{-x})^2}$

$y' = 0$ akko $4e^{-x}+3 = 0$

$\underbrace{e^{-x}}_{\substack{>0 \\ \forall x}} = \frac{-3}{4}$

$y' > 0$ za $\forall x \in R$

F-ja nema ekstrem i raste za $\forall x$

$y'' = \frac{e^{4x}(9e^{2x}+23e^{x}+16)}{(e^{x}+1)^3}$

$e^{x} = t$

$9t^2+23t+16 = 0$

$D = 529-576 < 0$

$y'' \neq 0$ za $\forall x \in R$

x	$(-\infty, +\infty)$
y''	$+$
y	$\cup$

tabela konveksnosti i konkavnosti

F-ja nema prevojnih tački.

Odrediti ekstreme, prevojne tačke te intervale konveksnosti i konkavnosti f-je $y = \frac{e^{2x}}{1+e^{2x}}$.

Rj. -upute:

$e^{2x} > 0 \quad \forall x \in R$

D.p. $x \in R$

$x \in (-\infty, +\infty)$

$$y' = \frac{2e^{2x}}{(1+e^{2x})^2}$$

$y' \neq 0 \quad \forall x \in R$

F-ja nema ekstrem

$y' > 0 \quad \forall x \in R$

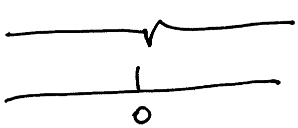
F-ja raste za $\forall x$.

$$y'' = -4\frac{e^{2x}(e^{2x}-1)}{(e^{2x}+1)^3}$$

$y'' = 0$ akko $\underbrace{e^{2x}}_{>0}(e^{2x}-1) = 0$

$e^{2x} = 1$

$x = 0$



prekidi y + nule y''

x	$(-\infty, 0)$	$(0, +\infty)$
y''	+	−
y	∪	∩

P.T.

tabela konveksnosti i konkavnosti

$$f(0) = \frac{1}{1+1} = \frac{1}{2}$$

$P.T.\left(0, \frac{1}{2}\right)$

(#) Odrediti ekstreme, prevojne tačke te intervale konveksnosti i konkavnosti f-je $y = \dfrac{e^{3x}}{1-x}$.

Rj. - upute:

D.p. $1-x \neq 0$

$x \neq 1$

$x \in (-\infty, 1) \cup (1, +\infty)$

$$y' = -\frac{e^{3x}(3x-4)}{(x-1)^2}$$

$y' = 0$ akko $3x-4=0$

$x = \frac{4}{3}$

$1 \qquad \frac{4}{3}$

← prekidi f-je y + nule y'

x	$(-\infty, 1)$	$(1, \frac{4}{3})$	$(\frac{4}{3}, +\infty)$
y'	+	+	−
y	↗	↗	↘

MAX

tabela rasta i opadanja

$$f\left(\frac{4}{3}\right) = \frac{e^4}{1-\frac{4}{3}} = -3e^4$$

$MAX\left(\frac{4}{3}, -3e^4\right)$

$$y'' = -\frac{e^{3x}(9x^2-24x+17)}{(x-1)^3}$$

$9x^2-24x+17=0$

$D = 576 - 612 < 0$

$9x^2-24x+17 > 0 \quad \forall x$

F-ja nema prevojnih tački

1

← prekidi y + nule y''

x	$(-\infty, 1)$	$(1, +\infty)$
y''	+	−
y	$\cup$	$\frown$

tabela konveksnosti i konkavnosti

#. Odrediti parametre a i b tako da f-ja

a) $y=\frac{(ax+b)^4}{x^3}$ ima kosu asimptotu u pravoj $y=x-4$

b) $y=\frac{(ax+b)^3}{x^2}$ ima kosu asimptotu u pravoj $y=27x+9$

c) $y=\frac{(ax+b)^2}{x}$ ima kosu asimptotu u pravoj $y=4x+4$

d) $y=\frac{a^2x^3+b^3x^2+1}{x^2}$ ima kosu asimptotu u pravoj $y=64x-27$

Rj.-upute

Kosa asimptota je u obliku $y=kx+n$ gdje je $k=\lim\limits_{x\to\infty}\frac{f(x)}{x}$, $n=\lim\limits_{x\to\infty}(f(x)-kx)$

a) $k=\lim\limits_{x\to\infty}\frac{f(x)}{x}=\lim\limits_{x\to\infty}\frac{(ax+b)^4}{x^4}=\lim\limits_{x\to\infty}\frac{a^4x^4+\dots}{x^4}=a^4$ $\quad a^4=1$

$a=1$

$$\begin{array}{c}1\\1\ 1\\1\ 2\ 1\\1\ 3\ 3\ 1\\1\ 4\ 6\ 4\ 1\end{array}$$

$n=\lim\limits_{x\to\infty}[f(x)-kx]=\lim\limits_{x\to\infty}\left[\frac{(x+b)^4}{x^3}-x\right]=\lim\limits_{x\to\infty}\frac{x^4+4bx^3+\dots-x^4}{x^3}=4b$

$4b=-4 \Rightarrow b=-1$ Traženi parametri su $a=1$, $b=-1$

b) $k=\lim\limits_{x\to\infty}\frac{(ax+b)^3}{x^3}=\lim\limits_{x\to\infty}\frac{a^3x^3+\dots}{x^3}=a^3 \Rightarrow a^3=27 \Rightarrow a=\sqrt[3]{27}=3$

$n=\lim\limits_{x\to\infty}\left[\frac{(3x+b)^3}{x^2}-27x\right]=\lim\limits_{x\to\infty}\frac{27x^3+9x^2b+\dots-27x^3}{x^2}=9b \Rightarrow$ $9b=9$

$b=1$

c) $k=\lim\limits_{x\to\infty}\frac{(ax+b)^2}{x^2}=\lim\limits_{x\to\infty}\frac{a^2x^2+2abx+b^2}{x^2}=a^2 \Rightarrow a^2=4 \Rightarrow a=2$

$n=\lim\limits_{x\to\infty}\left[\frac{(2x+b)^2}{x}-4x\right]=\lim\limits_{x\to\infty}\frac{4x^2+4bx+b^2-4x^2}{x}=4b \Rightarrow$ $4b=16$

$b=4$

d) $k=\lim\limits_{x\to\infty}\frac{a^2x^3+b^3x^2+1}{x^3}=a^2 \Rightarrow a^2=64 \Rightarrow a=8$

$b=-3$

$n=\lim\limits_{x\to\infty}\left[\frac{64x^3+b^3x^2+1}{x^2}-64x\right]=\lim\limits_{x\to\infty}\frac{b^3x^2+1}{x^2}=b^3 \Rightarrow b^3=-27$

Odrediti ekstreme f-je $z = x^2 + y^3 + 4x\sqrt{x^3} - 3y$.

Rj.-upute

$$\frac{\partial z}{\partial x} = 2x + 10\sqrt{x^3}$$

$$\frac{\partial z}{\partial y} = 3y^2 - 3$$

$$2x + 10x\sqrt{x} = 0$$

$$3y^2 - 3 = 0$$

⋮

Stacionarne tačke su

$M(0;1)$ i $N(0;-1)$

$$\frac{\partial^2 z}{\partial x^2} = 15\sqrt{x} + 2$$

$$\frac{\partial^2 z}{\partial x \partial y} = 0$$

$$\frac{\partial^2 z}{\partial y^2} = 6y$$

Za $M(0;1)$

$$D = \begin{vmatrix} A & B \\ B & C \end{vmatrix} = 12$$

$$z_{MAX}(0;1) =$$

Za $N(0;-1)$ imamo $D = -12$

$D < 0$

U tački N f-ja nema ekstrem

(#) Odrediti ekstreme f-je $z = 3\ln\frac{x}{6} + \ln(12-y-x) + 2\ln y$

Rj.-upute

$$\frac{\partial z}{\partial x} = \frac{1}{x+y-12} + \frac{3}{x} \qquad \frac{1}{x+y-12} + \frac{3}{x} = 0$$

$$\frac{\partial z}{\partial y} = \frac{1}{x+y-12} + \frac{2}{y} \qquad \frac{1}{x+y-12} + \frac{2}{y} = 0$$

$\vdots$

Stacionarna tačka je $M(6;4)$

$$\frac{\partial^2 z}{\partial x^2} = -\frac{1}{(x+y-12)^2} - \frac{3}{x^2}$$

$$\frac{\partial^2 z}{\partial x \partial y} = -\frac{1}{(x+y-12)^2}$$

$$\frac{\partial^2 z}{\partial y^2} = -\frac{1}{(x+y-12)^2} - \frac{2}{y^2}$$

Za $M(6;4)$ imamo

$A = -\frac{1}{3}$

$B = -\frac{1}{4}$

$C = -3/8$

$D = \begin{vmatrix} A & B \\ B & C \end{vmatrix} = \frac{1}{16} > 0$

$$z_{MAX}(6;4) = 3\ln 1 + \ln 2 + 2\ln 4 = \ln 2 + 2\ln 4$$

$$= \ln 2 + \ln 4^2 = \ln 32$$

Odrediti ekstreme f-je $z = x^3 + y^2 - 3x + 4\sqrt{y^5}$.

Rj. -upute

$$\frac{\partial z}{\partial x} = 3x^2 - 3$$

$$\frac{\partial z}{\partial y} = 2y + \frac{10y^4}{\sqrt{y^5}} = 2y + \frac{10y^2}{\sqrt{y}} = 2y + 10y\sqrt{y} = 2y + 10\sqrt{y^3}$$

$$3x^2 - 3 = 0$$
$$y(2 + 10\sqrt{y}) = 0$$

$\vdots$

Stacionarne tačke su
$M(1;0)$ i $N(-1;0)$

$$\frac{\partial^2 z}{\partial x^2} = 6x$$

$$\frac{\partial^2 z}{\partial x \partial y} = 0$$

$$\frac{\partial^2 z}{\partial y^2} = 15\sqrt{y} + 2$$

Za $M(1;0)$

$$D = \begin{vmatrix} A & B \\ B & C \end{vmatrix} = 12$$

$$z_{MAX}(1;0) = -2$$

Za $N(-1;0)$

$$D = -12 < 0$$

U tački N f-ja nema ekstrema.

(#) Odrediti ekstreme f-je $z = 2\ln x + \ln(12-x-y) + 3\ln\frac{y}{6}$

Rj.-upute

$$\frac{\partial z}{\partial x} = \frac{1}{x+y-12} + \frac{2}{x}$$

$$\frac{\partial z}{\partial y} = \frac{1}{x+y-12} + \frac{3}{y}$$

$$\frac{1}{x+y-12} + \frac{2}{x} = 0$$

$$\frac{1}{x+y-12} + \frac{3}{y} = 0$$

$\vdots$

Stacionarna tačka je $N(4;6)$

$$\frac{\partial^2 z}{\partial x^2} = -\frac{1}{(x+y-12)^2} - \frac{2}{x^2}$$

$$\frac{\partial^2 z}{\partial x \partial y} = -\frac{1}{(x+y-12)^2}$$

$$\frac{\partial^2 z}{\partial y^2} = -\frac{1}{(x+y-12)^2} - \frac{3}{y^2}$$

Za $N(4;6)$

$A = -\frac{3}{8}$, $B = -\frac{1}{4}$, $C = -\frac{1}{3}$

$D = \frac{1}{16} > 0$

$$z_{MAX}(4;6) = \ln 32$$

Odrediti sljedeće integrale

a) $\displaystyle\int \frac{\sqrt[6]{x+1}\,dx}{\sqrt{x+1}+\sqrt[3]{x+1}}$ b) $\displaystyle\int \frac{dx}{3x-4\sqrt{x}}$

c) $\displaystyle\int \frac{\sqrt{x}\,dx}{\sqrt[4]{x^3}+4}$ d) $\displaystyle\int \frac{\sqrt{x}\,dx}{\sqrt[3]{x^2}-\sqrt[4]{x}}$

Rj.:

a) $$\int \frac{\sqrt[6]{x+1}}{\sqrt{x+1}+\sqrt[3]{x+1}}\,dx = \left| \begin{array}{c} NZS(1,3,6)=6 \\ x+1=u^6 \\ dx=6u^5du \end{array} \right| = \int \frac{u}{u^3+u^2}\,6u^5\,du =$$

$$=6\int \frac{u^4}{u+1}\,du \overset{(*)}{=} 6\int \left(u^3-u^2+u-1+\frac{1}{u+1}\right)du \overset{(1)}{=}$$

$u^4:(u+1)=u^3-u^2+u-1$
$-\ u^4+u^3$
$-u^3$
$-\ -u^3-u^2$
u^2
$-\ u^2+u$
$-u$
$-\ -u-1$
1

$$\frac{u^4}{u+1}=u^3-u^2+u-1+\frac{1}{u+1}$$

$\ldots(*)$

$$\overset{(1)}{=} \frac{6}{4}u^4-\frac{6}{3}u^3+\frac{6}{2}u^2-6u+6\ln|u+1|+C=$$

$$=\frac{3}{2}\sqrt[3]{(x+1)^2}-2\sqrt{x+1}+3\sqrt[3]{x+1}-\sqrt[6]{x+1}+6\ln\left|\sqrt[6]{x+1}+1\right|+C$$

b) $$\int \frac{dx}{3x-4\sqrt{x}} = \left| \begin{array}{c} x=u^2 \\ dx=2u\,du \end{array} \right| = \int \frac{2u\,du}{\underbrace{3u^2-4u}_{u(3u-4)}} = \frac{2}{3}\int \frac{du}{u-\frac{4}{3}} =$$

$$\frac{2}{3}\int \frac{d(u-\frac{4}{3})}{u-\frac{4}{3}} = \frac{2}{3}\ln|u-\frac{4}{3}|+C_1 = \frac{2}{3}\ln|\sqrt{x}-\frac{4}{3}|+C_1 =$$

$$\frac{2}{3}\ln|\frac{3\sqrt{x}-4}{3}|+C_1 = \frac{2}{3}\ln|3\sqrt{x}-4|+C$$

c) $$\int \frac{\sqrt{x}\,dx}{\sqrt[4]{x^3}+4} = \int \frac{x^{\frac{1}{2}}}{x^{\frac{3}{4}}+4}dx = \left| \begin{array}{c} NZS(3,4)=4 \\ x=u^4 \\ dx=4u^3du \end{array} \right| =$$

$$=4\int \frac{u^2}{u^3+4}u^3du = 4\int \frac{u^5}{u^3+4} = 4\int \left(u^2-\frac{4u^2}{u^3+4}\right)du =$$

$$\begin{array}{l} u^5:(u^3+4)=u^2 \\ \underline{u^5+4u^2} \\ \quad -4u^2 \end{array}$$

$$=4\int u^2du - 16\cdot\frac{1}{3}\int\frac{d(u^3+4)}{u^3+4} =$$

$$=\frac{4}{3}u^3-\frac{16}{3}\ln|u^3+4|+C =$$

$$=\frac{4}{3}\sqrt[4]{x^3}-\frac{16}{3}\ln|\sqrt[4]{x^3}+4|+C$$

d) $$\int \frac{\sqrt{x}\,dx}{\sqrt[3]{x^2}-\sqrt[4]{x}} = \left| \begin{array}{c} NZS(2,3,4)=12 \\ x=u^{12} \\ dx=12u^{11}du \end{array} \right| = \int \frac{u^6}{u^8-u^3}12u^{11}du =$$

$$=12\int \frac{u^{14}}{u^5-1}du = 12\int\left(u^9+u^4+\frac{u^4}{u^5-1}\right)du =$$

$$\begin{array}{l} u^{14}:(u^5-1)=u^9+u^4 \\ \underline{u^{14}-u^9} \\ \quad u^9 \\ \underline{\;-u^9-u^4} \\ \qquad u^4 \end{array}$$

$$=12\cdot\frac{u^{10}}{10}+12\frac{u^5}{5}+12\cdot\frac{1}{5}\int\frac{d(u^5-1)}{u^5-1} =$$

$$=\frac{6}{5}\left(x^{\frac{1}{12}}\right)^{10}+\frac{12}{5}\left(x^{\frac{1}{12}}\right)^5+\frac{12}{5}\ln|(x^{\frac{1}{12}})^5-1|+C$$

$$=\frac{6}{5}\sqrt[6]{x^5}+\frac{12}{5}\sqrt[12]{x^5}+\frac{12}{5}\ln|\sqrt[12]{x^5}-1|+C.$$